EML 4312: Control of Mechanical Engineering Systems

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History and Background Material

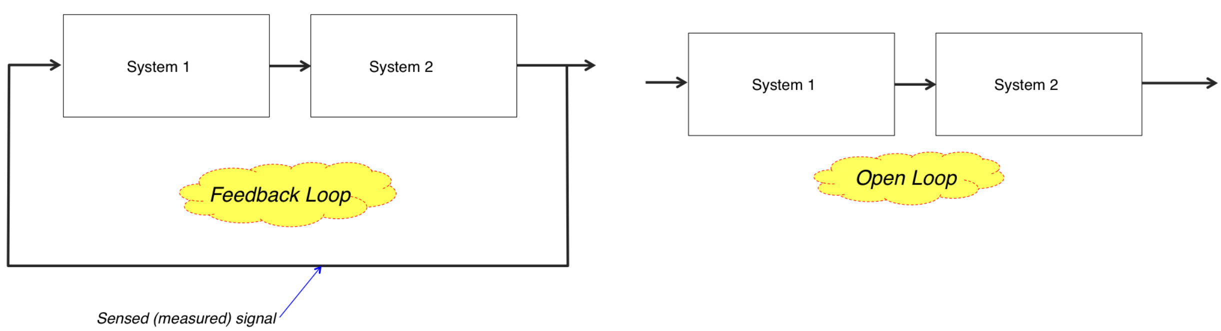
Preliminaries

Various notions of feedback

- *Åström & Murray, Feedback Systems, Chapter 1...*

  "The term *feedback* refers to a situation in which two (or more) dynamical systems are connected together such that each system influences the other and their dynamics are thus strongly coupled. Simple causal reasoning about a feedback system is difficult because the first system influences the second and the second system influences the first, leading to a circular argument. This makes reasoning based on cause and effect tricky, and it is necessary to analyze the system as a whole. A consequence of this is that the behavior of feedback systems is often counterintuitive, and it is therefore necessary to resort to formal methods to understand them."

  An illustration of the above; in particular, the distinction between *feedback* and *open-loop* systems is shown in Fig.1.



(a) Feedback system: **closed loop**

(b) **Open loop** system

Figure 1: Closed loop and open loop: an illustration of interconnections as described by Åström & Murray

- *Doyle, Francis & Tannenbaum, Feedback Control Theory, Chapter 1...*

  "Without control systems, there could be no manufacturing, no vehicles, no computers, no regulated environment - in short, no technology. Control systems are what make machines, in the broadest sense of the term, function as intended. Control systems are most often based on the principle of feedback, whereby the signal to be controlled is compared to a desired reference signal and the discrepancy used to compute corrective control action..."

  ..also from Chapter 3... "The most elementary feedback control system has three components: a plant (the object to be controlled, no matter what it is, is always called the *plant*, a sensor to measure the output of the plant, and a controller to generate the plant's input. Usually actuators are lumped in with the plant."

- *Ogata, Feedback Systems, Chapter 1...*

  "A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a *feedback control system*... Feedback control systems are not limited to engineering but can be found in various nonengineering fields as well. The human body, for instance, is a highly advanced feedback control system. Both body temperature and blood pressure are kept contact by means of physiological feedback.."

  "..Feedback control systems are often referred to as *closed-loop control systems*. In practice, the terms feedback control and closed-loop control are used interchangeably In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal (which may be the output signal itself or a function of the output signal and its derivatives and/or integrals), is fed to the controller so as to reduce the error and bring the output of the systems to a desired value... ..Those systems in which the output has no effect on the control action are called *open-loop control systems*. In other words, in an open-loop control system the output is neither measured nor fed back for comparison with the input. "

- *Dorf & Bishop, Modern Control Systems, Chapter 4...*

  "A control system is defined as an interconnection of components forming a system that will provide a desired system response. Because this desired system response is known, a signal proportional to the error between the desired and the actual response is generated. The use of this signal to control the process results in a closed-loop sequence of operations that is called a feedback system."

You can probably gather from above that feedback systems involve an interconnection among the following key elements:

i.) a **process** (to be controlled). Commonly also referred to as the *plant*, this is the "central object" - the entity that must be controlled, i.e. made to behave a certain way.

ii.) **sensors.** These are what give feedback systems their "awareness" by virtue of the measurements they make to be used as feedback. Without sensors there can be no feedback.

iii.) **controller.** This is the computer, or the algorithm, to which we feed the difference between the measured signal and the externally input reference signal, i.e. the *error signal*:

$$\underbrace{e(t)}_{\text{error}} = \underbrace{y(t)}_{\text{measurement}} - \underbrace{r(t)}_{\text{input (reference)}} \tag{1}$$

The controller uses the error signal to generate control commands. Depending on how the control is generated from the error, we get different types of controllers, e.g. see PID controller in Eqs.(3) below.

iv.) **actuators.** These are the elements that *physically implement* the control commands. In mathematical modeling, actuators are often clubbed together with the controller.

♣ The composition of the above systems in feedback fashion is called a *feedback control system*: see Fig.(2). Note that the measured signal is *subtracted* from the reference signal: this is called *negative feedback*. Inadvertent positive feedback often leads to poor performance, even instability.

♣ A diagram of the type shown in Fig.(2) is called a *block diagram*. In practice, it is common to combine the control & actuator blocks into a single block with input $e(t)$ and output $u(t)$ (control signal). This is also true in actual, physical control systems: the actions of one or several of these blocks may be performed by a single component. See examples below.

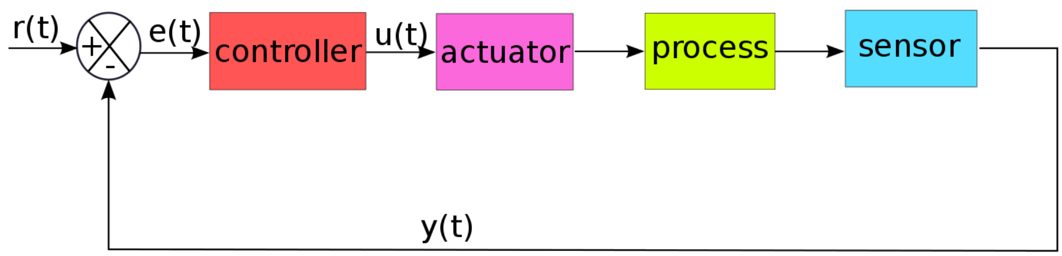


Figure 2: A feedback control system, a.k.a. closed-loop system

Examples of Feedback Control

Feedback is ubiquitous, both in man-made and natural systems. See the introductory presentation for a wide range of examples. Here, we just consider two instances of feedback in man-made systems to illustrate the above concepts.

A.) Automobile cruise control. The idea is to maintain a fixed speed automatically (i.e. without continuous participation of the driver) despite undulations in the road or other disturbances. The reference signal is simply the desired speed at which the driver wants the car to operate. It is input using the cruise control buttons in the car. The sensor here is a car speed measurement device, i.e. a tachometer. The difference between the actual (i.e. sensed) and desired (i.e. reference) speeds is provided to the cruise control computer (a piece of circuitry as shown) that analyses the signal and generates control commands for the actuator. The actuator in the present case is simply the engine. More specifically, the actuator is the device that controls the fuel injection to the engine: if the plant (car) is going slower than the reference speed, fuel flow is increased. Similarly, fuel flow is lowered if the car is going faster than desired, e.g. perhaps because the car is going on a downhill road.

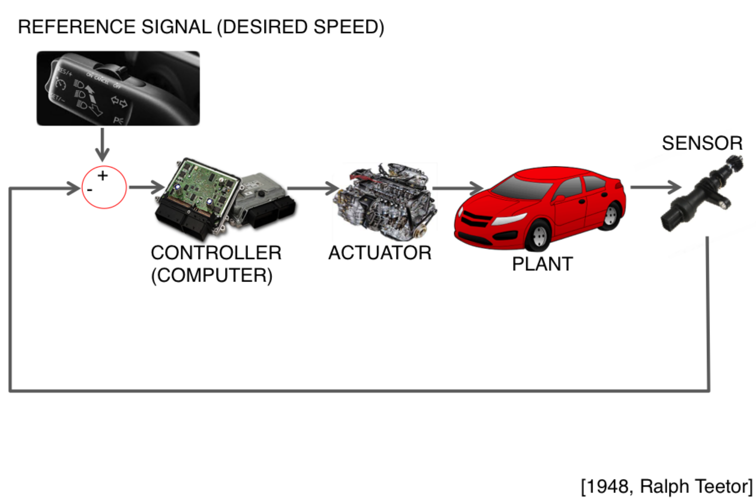


Figure 3: Feedback example 1: automobile cruise control

Note there is a “problem” with the above illustration: the “plant”, which is the car, actually contains within it the controller, actuator and the sensor. So, you may think, in the figure above, the picture of the car is meant to represent everything *except* these elements. This is not exactly true, because the sensor, controller and actuator move *with* the plant, and must also be controlled, e.g. their mass (inertia) contributes to the computation of the control signal. So, a better way is to think of the entire feedback system as a single cohesive unit containing among other things, controlling, actuating and sensing elements that functions harmoniously in a feedback interconnection.

B.) Home temperature control. The idea is to maintain a fixed temperature inside the house (or a pre-programmed variation, e.g. 78 deg between 9 AM and 5 PM and 72 deg between 5 PM and 9 AM). The house receives disturbance inputs from the external environment that try to drift the system away from the desired program (Fig.(4)). The thermostat in this case is both the sensor and the controller, because it measures the current temperature and issues the control signal to make corrections. In fact, it is also the place where the reference (desired) signal is provided by the inhabitants. The actuator is the ac unit that regulates the air flow in the ducts.

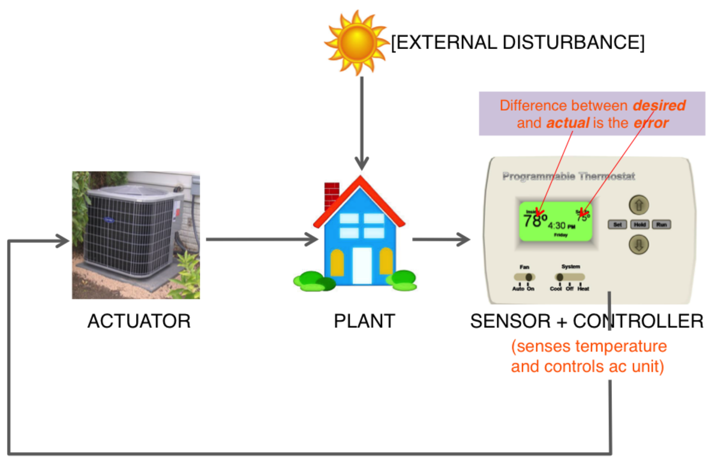


Figure 4: Feedback example 2: home temperature control

Properties of Feedback

Feedback is used extensively in a variety of engineering contexts (and beyond!). Its principle is simple: provide corrective actions ($\sim$ control) based on the difference between desired and actual (determined by sensor) performance. Feedback has been known to drastically improve system capability. Below we enumerate some of its key features.

Advantages of Feedback

1. The main use of feedback is to provide **robustness to uncertainty**. The idea is that by measuring the difference between the sensed value of a regulated signal and its desired value, a corrective action can be supplied. This difference is often referred to as an *error signal*:

$$e(t) \;=\; y_s(t) - y_d(t) \tag{2}$$

where, the actual value of the regulated signal is $y(t)$. Its measured (sensed) value is $y_s(t)$, which is different from the actual signal due to sensor (instrument) error that typically manifests in the form of measurement noise. The desired value is $y_d(t)$, often also written as $y_r(t)$, and called the *reference signal.* The widely popular PID controller is the summation of three control measures – (i) proportional (P) to the error signal, (ii) integral of the error signal, and (iii) derivative of the error signal:

$$u_P(t) = K_P e(t) \tag{3a}$$

$$u_I(t) = K_I \int_0^t e(\tau) d\tau \tag{3b}$$

$$u_D(t) = K_D \frac{de}{dt}(t) \tag{3c}$$

$$u_{PID}(t) = u_P(t) + u_I(t) + u_D(t) \tag{3d}$$

Systems are invariably driven by forces that are poorly understood, e.g. wind gusts hitting an airplane, bumps on a road affecting the operation of a car, etc. These forces are best modeled as "random" or "uncertain" disturbance inputs that perturb the system under study. In addition, there are sources of uncertainty within the system model, e.g. parameters whose true value may be different from their assumed value. Consider the numerous components of an electrical circuit, whose operational values can differ vastly from their assumed values and are often a function of the operating conditions, e.g. the impedance of a circuit element may depend on its temperature. Despite these intrinsic and extrinsic uncertainties, we want our system to behave in a certain way, in the sense of achieving certain well-defined metrics of "performance". Feedback, perhaps in the form of a PID controller of Eq.(3), allows us to fulfill this objective, thereby making the system robust against internal as well as external uncertainties.

2. An important use of feedback is to ***fundamentally* alter the dynamical behavior** of a system. Unstable systems (such as an inverted pendulum, or one of its more physical realizations: a powered rocket!) can be stablized, systems with sluggish response can be made agile (e.g. the *stiffness* of an aircraft can be reduced), systems with drifting operating points can be held constant to operate in a desired region within its performance envelope, etc. This feature of feedback is sometimes called *design of dynamics* (Åström & Murray). Such "dynamics design" also serves to increase modularity of the overall system. By essentially controlling the system to have a desired overall dynamic profile, we can mask the complexities and variability in its subsystems, precluding the need to tune each individual such subsystem to achieve desired behavior.

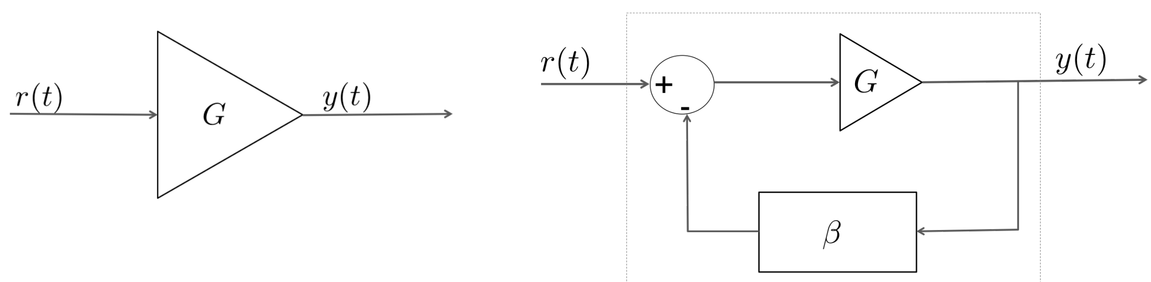
3. Feedback, in recent years, has helped up achieve unparalleled **autonomy** of dynamical systems. For a system (called *agent* henceforth) to operate autonomously, i.e. unsupervised by humans, it must have so-called situational awareness and decision making capability in a potentially unknown, unstructured environment. This invokes several streams of inquiry that are usually encountered in the community of artificial intelligence, e.g. learning, adaptation, and even abstract reasoning, all with some sense of optimality. There is an increasing role of dynamics, robustness and interconnection in these fields, leading to new branches of control such as distributed control and cooperative control involving multi-agent autonomous teams. A commonly cited example is autonomous cars, which are now reaching a reasonable level of maturity, e.g. the vehicles that participate in the DARPA Grand Challenge, including UF's NaviGATOR.

❢ **Example** *Black's amplifier.* Harold S. Black is credited with the invention of the negative feedback amplifier in 1927, working as an engineer in the Bell Labs. As he recounts in his 1977 paper, *Inventing the negative feedback amplifier* (IEEE Spectrum (14), 1977, pp. 55-60):

"Few rosier dreams could be dreamed than that of an amplifier whose overall performance is perfectly constant, and in whose output distortion constitutes only one-hundred millionth of total energy, although the component parts may be far from linear in their response and their gain may vary over a considerable range. But the dreamer who awakes in amazement to find that such an amplifier can be built has additional surprises in store for him. These benefits can be obtained by simply throwing away some gain, and by utilizing feedback action....

...Then came the morning of Tuesday, August 2, 1927, when the concept of the negative feedback amplifier came to me in a flash while I was crossing the Hudson River on the Lackawanna Ferry, on my way to work. For more than 50 years I have pondered how and why the idea came, and I can't say any more today than I could that morning. All I know is that after several years of hard work on the problem, I suddenly realized that if I fed the amplifier output back to the input, in reverse phase, and kept the device from oscillating (singing, as we called it then), I would have exactly what I wanted: a means of canceling out the distortion in the output. I opened my morning newspaper and on a page of The New York Times I sketched a simple canonical diagram ... when I reached the laboratory at 463 West Street, it was witnessed, understood, and signed by the late Earl C. Blessing."

Black's amplifier is probably the simplest, yet a most outstanding example of how feedback makes for robust performance (steady in the face of uncertainty or variability) and also, how one can make "good systems from bad components" (Åström). When he first implemented his now ubiquitous feedback design, amplifiers were used in the field of telephone communications to compensate for signal attenuation (~ weakening) over long lines. Vacuum tubes were the most popular for construction of such amplifiers, but were notorious for causing signal distortion due to nonlinear characteristics of the tube. Black employed feedback to make the amplifier insensitive to variations in tube characteristics (i.e. he made the amplifier *robust*). There was a price to pay – the overall amplification achieved was now much lower. The benefit however, namely reliability in the face of uncertainty, easily outweighed the loss. To appreciate what the Black's feedback amplifier does, consider first the "open loop amplifier" shown on the left in Fig.5(a). In the 1920's, the circuitry behind the "G" block consisted of vacuum tubes, which behaved erratically as mentioned above.



(a) Open loop amplifier (b) Negative feedback amplifier designed by Harold Black

Figure 5: Black's feedback amplifier ensures robustness despite variability in component characteristics

Clearly, the relationship between the input, $r(t)$, and output signal, $y(t)$ is

$$y(t) \; = \; G\, r(t) \tag{4}$$

where, G is a constant ***gain***, i.e. the amplification in the signal. As operating conditions change, vacuum tubes misbehave, taking with them the numerical value of G. If G was nominally intended to provide a hundred-fold amplification, it is not unrealistic to consider 50% variability, meaning that the actual amplification would be anything between 50 and 150, i.e. $G_{\text{ol}} = G \in [50, 150]$, where the subscript "ol" stands for open-loop.

On the other hand, note that the input-output relationship for the triangular box shown in Fig.(5(b)) is:

$$y(t) \; = \; G\,(r(t) - \beta y(t)) \tag{5}$$

such that, we have the relationship between $r(t)$ and $y(t)$ as

$$y(t) \; = \; \underbrace{\frac{G}{1+\beta G}}_{G_{\text{cl}}} \; r(t) \tag{6}$$

whereby, we can identify the ***closed-loop gain*** as $G_{\text{cl}} \; = \; G/(1+\beta G)$: this is the net amplification of the input signal.

As before, the triangular box is made of vacuum tubes such that its nominal value is $G = 100$, but in reality it can be anything between 50 and 150 (50% variability). The variable β is thought of as "feedback gain" and it is the factor of the measured signal that is used for generating the error signal. When $\beta = 1$, we say we have *unity feedback*.

• Recall that the "price to pay" for feedback is a reduction in amplification. This is apparent now, comparing Eqs.(4) and (6):

$$G_{\text{ol}} = G$$
$$G_{\text{cl}} = \frac{G}{1 + \beta G}$$

Since $(1 + \beta G) > 1$, we have $G_{\text{cl}} < G_{\text{ol}}$! That's the downside. Now, lets see why feedback is good. Table (1) illustrates the closed-loop gain for four values of β (0.1, 0.2, 0.5, & 1.0 i.e. unity feedback). For each case, three values of G are considered, $G = 50, 100, 150$, which represents its nominal, and lower/upper extremes, capturing a 50% variability. Clearly, variation in the open-loop gain is the same. However in each case, the variation in G_{cl} is much lower! You can see that as β is increased, the overall closed loop gain declines, but so does the variation. In fact, for unity feedback, there is no "gain", there's attenuation. The full variation of G_{cl} versus G is shown in Figs.(6). Also noted here for each case is the mean value of G_{cl} and its maximum variation above and below the mean. As a designer you must decide: what would you rather have; more amplification with greater variability, or less amplification with greater reliability? In other words, where is the sweet spot that strikes the best balance vis-à-vis design requirements?

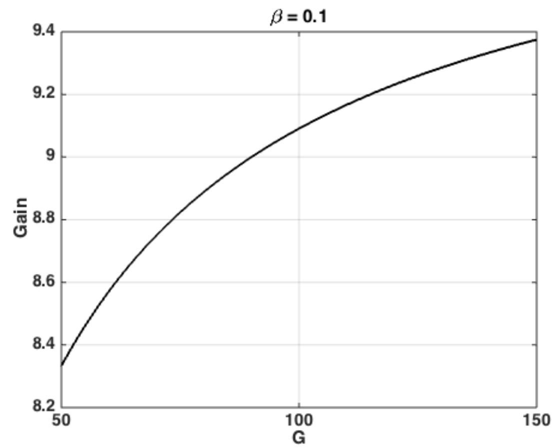
Table 1: Closed loop gain under component variability: Black's amplifier

G	β	G_{cl}	G_{ol}
100	0.1	9.09	100
50	0.1	8.33	50
150	0.1	9.38	150
100	0.2	4.76	100
50	0.2	4.55	50
150	0.2	4.84	150
100	0.5	1.96	100
50	0.5	1.92	50
150	0.5	1.97	150
100	1	0.99	100
50	1	0.98	50
150	1	0.99	150

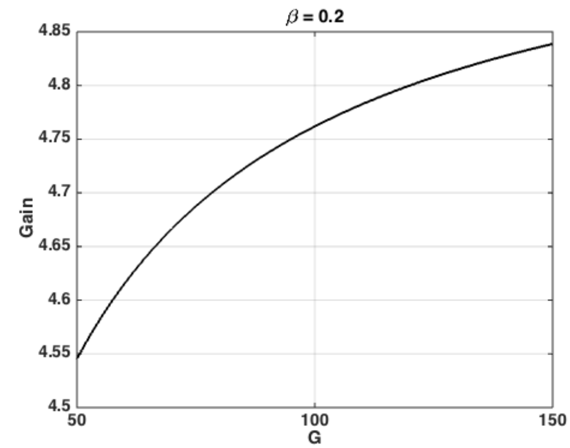
• The modern-day realization of Black's amplifier is the *operational amplifier* ("op-amp").

❣ **Example** *Simplified cruise control.* Consider the simplified cruise control block model shown in Fig.(7). Suppose the desired speed (reference signal) is $r(t) = 60$ mph. A *disturbance signal*, $w(t)$ is shown that perturbs the plant (car), and we assume that it is a constant, $w(t) = -3$ mph (perhaps an uphill road). In reality, the disturbance is impossible to predict and we know it here only because this is a "toy" problem. Unity feedback is shown, such that "all of the measured signal" is used for construction of the error signal. Also for simplicity, we assume that the plant (P) adds no dynamics, i.e. it simply transmits the input signal as output. We have the cases of open-loop & closed-loop control shown below:

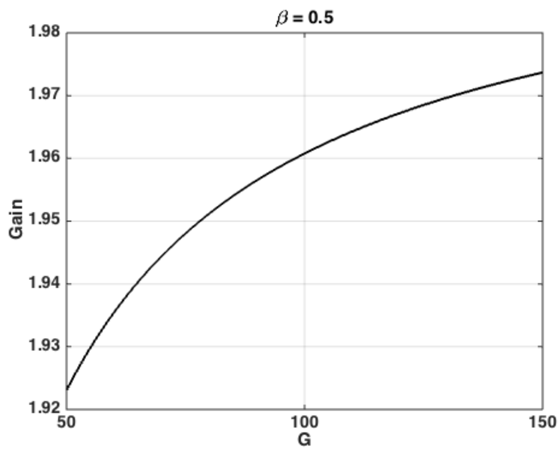
- Open-loop Control: There is no feedback, which can also be understood as setting $\beta = 0$! Also, there is no reason for the controller to modify the input signal... for the reason shown below in the following



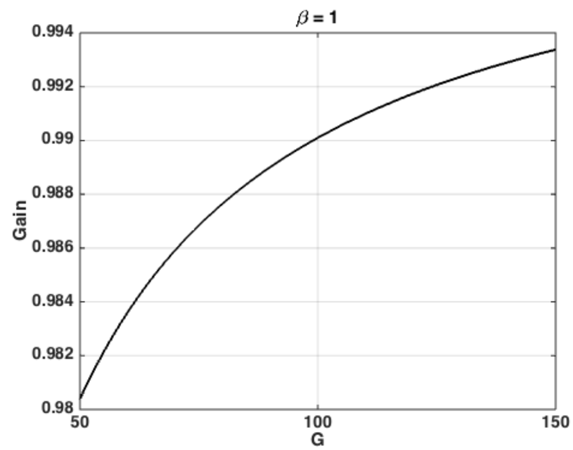
(a) G_{cl}: mean: 9.02, max up: 3.96%, max down: 7.59%



(b) G_{cl}: mean: 4.74, max up: 2.07%, max down: 4.11%



(c) G_{cl}: mean: 1.96, max up: 0.85%, max down: 1.73%



(d) G_{cl}: mean: 0.99, max up: 0.43%, max down: 0.88%

Figure 6: Variation in closed loop gain as a function of G: four cases.

algebra:

$$e(t) = r(t) - \beta y(t) = r(t) \quad (\beta = 0) \tag{7a}$$

$$u(t) = e(t) \tag{7b}$$

$$y_{\text{ol}}(t) = u(t) + w(t) \tag{7c}$$

Clearly, the output is at the mercy of the disturbance input:

$$y_{\text{ol}}(t) = r(t) + w(t) \tag{8}$$

which, in this case turns out to be 57 mph.

- Closed-loop Control: We will use unity feedback, and use *proportional control*, i.e. $u(t) = ke(t)$:

$$u(t) = ke(t) = k(r(t) - \beta y(t)) \tag{9a}$$

$$y(t) = u(t) + w(t) \text{ such that,} \tag{9b}$$

$$= k(r(t) - \beta y(t)) + w(t) \tag{9c}$$

$$\Rightarrow y_{\text{cl}}(t) = \frac{k}{1+\beta k} r(t) + \frac{1}{1+\beta k} w(t) \tag{9d}$$

Table 2: Cruise Control Performance using Proportional Unity Feedback & Comparison with Open-loop Control

reference: $r(t)$	disturbance: $w(t)$	control gain: k	CL output: $y_{\text{cl}}(t)$	OL output: $y_{\text{ol}}(t)$
60	-3	0.1	2.7	57
60	-3	1	28.5	57
60	-3	10	54.3	57
60	-3	100	59.4	57
60	-5	100	59.4	55
60	+5	100	59.5	65
60	-10	100	59.3	50
60	+10	100	59.5	70

Table (2) makes for fascinating reading. When the feedback gain (k) is low, closed loop performance is bad! (we say the controller is *improperly tuned*) However, as the gain is increased, the closed-loop output starts tracking the reference signal better. In fact, for $k = 100$, you see that it doesn't really matter how bad the disturbance is! The closed-loop tracking is incredibly good, whereas as stated above, the open-loop performance is at the mercy of the disturbance. As you can see, in this case, the difference between carefree driving and getting a ticket is a well-tuned feedback controller!!

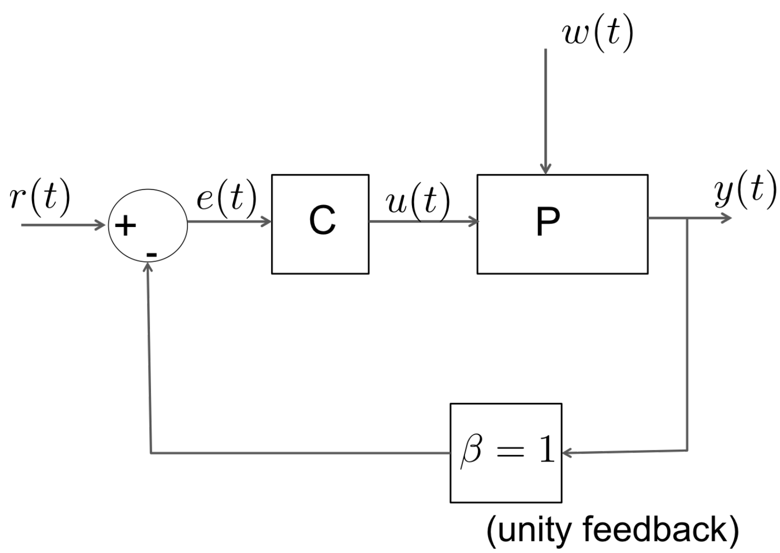


Figure 7: Simplified Cruise Control: Block Diagram

Disadvantages of Feedback

1. Feedback couples various parts of a system thereby increasing system **complexity**, especially in the sense of Åström & Murray: see "loss of intuition" above. More importantly, an improperly designed feedback system can become unstable, invariably due to so-called *positive-feedback*. In fact, positive feedback may be enough to destabilize a system that is otherwise stable.

2. Feedback increases system **vulnerability**, because the failure of a sensor (which is critical for feedback operation!) invariably entails failure of the entire system. Moreover, introduction of sensors allows injection of measurement noise into the system. As a result, sensors must be used with a *filter* to block out such noise and ensure that the system does not respond to it, possibly causing instability.

3. Demands *careful design*: because of the two drawbacks listed above, a feedback control system must be very carefully designed. The cost of new system components (sensors, computers and actuators) can vary from the very cheap to outlandishly expensive. For example, the 3D lidar sensors used in Google's autonomous car provide outstanding situational awareness and come at a hefty cost of about $75,000 a piece.

Feedforward Control

The discussion above suggests that feedback is reactive, in the sense that there must first be an error signal, which drives the control action. In some circumstances, it is possible however to estimate the disturbance before it enters the system, allowing us to take action before the system is driven off course. This control paradigm is known as *feedforward*. Feedforward is especially useful in shaping system response to commanded signals (a.k.a reference signals) because they are always known. A drawback of the F/F approach is that it requires an accurate model of the process (the plant), the absence of which can often lead to incorrect magnitude and/or timing of the control actions.

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