Problem 1

Consider a spring-mass system, with stiffness 4,000 N/m and mass of 10 kg subject to a harmonic force $F(t) = 400\cos 10t$ N. Find and plot the total response of the system using MatLab (print your code and plots) under the following initial conditions:

a. $x_0 = 0.1$ m, $\dot{x}_0 = 0$

b. $x_0 = 0$, $\dot{x}_0 = 10$ m/s

c. $x_0 = 0.1$ m, $\dot{x}_0 = 10$ m/s

$$y_a(t) = A_1\cos(w_n t) + A_2\sin(w_n t) + \frac{F_0}{w_n^2 - w^2}\cos(wt) \; \{t \geq 0\}$$

$x_0 = 0.1$

$v_0 = 0$

$A_1 = x_0 - \frac{F_0}{w_n^2 - w^2}$ $\quad A_1 = -1.23333333333$

$A_2 = v_0$ $\quad A_2 = 0$

$w_n = \sqrt{\frac{k}{m}}$ $\quad w_n = 20$

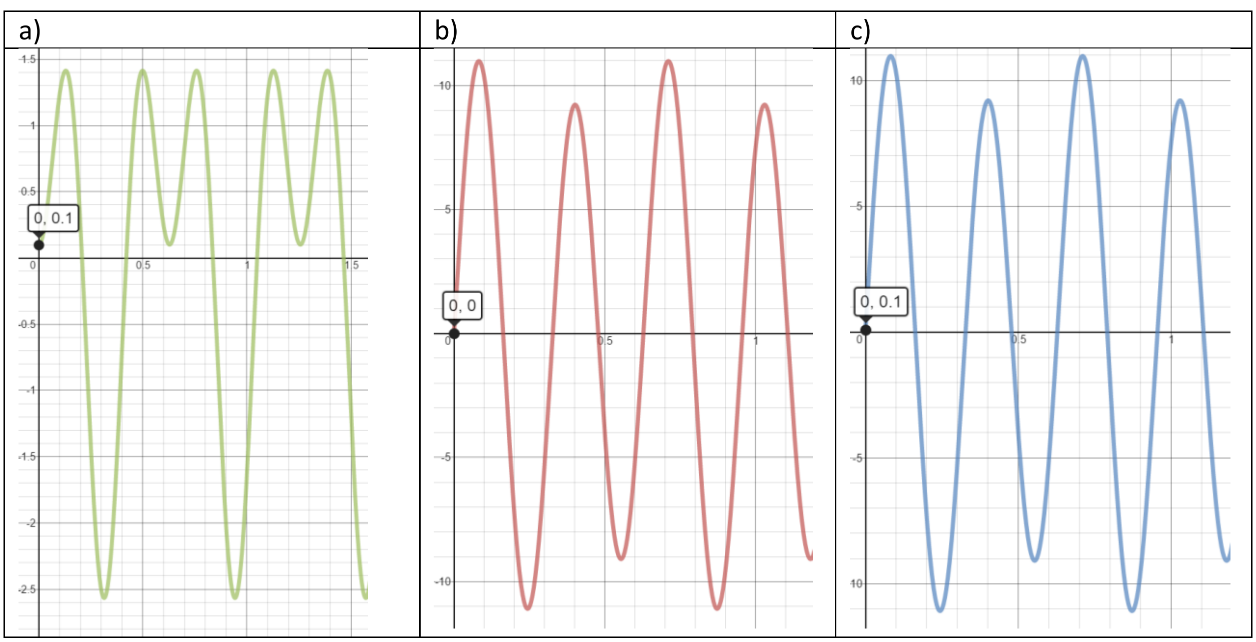
$F_0 = 400$

$w = 10$

$k = 4000$

$m = 10$

a)



Problem 2

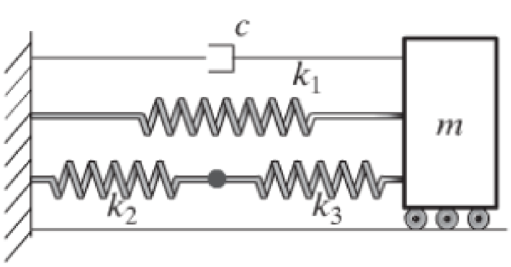
A spring-mass system is subjected to a harmonic force whose frequency is close to the natural frequency of the system. If the forcing frequency is 39.8 Hz and the natural frequency is 40.0 Hz, determine the period of beating.

$$\omega_{BEAT} = \omega_n - \omega = 40 - 39.8 = 0.2\ Hz$$

$$T_{BEAT} = \frac{2\pi}{\omega_{BEAT}} = 10\pi\ [s]$$

Problem 3

Calculate the natural frequency and damping ratio for the system shown below given the values m=10kg, c=100kg/s, $k1$ =4000N/m, $k2$ =200N/m and $k3$ = 1000 N/m. Assume that no friction acts on the rollers. Is the system overdamped, critically damped or underdamped?



Step 1 find the k_{eq}

$$k_{eq} = \frac{k_2 k_3}{k_2 + k_3} + k_1 = \frac{200 * 1000}{200 + 1000} + 4000 = 4166.66$$

Step 2 find ζ

$$\zeta = \frac{c}{2m\omega_n} = \frac{c}{2m\sqrt{\frac{k_{eq}}{m}}} = \frac{100}{2 \cdot 10 \cdot \sqrt{\frac{4166.66}{10}}} = 0.2449$$

Because ζ is less than 1 the system is underdamped

Problem 4

Derive the undamped forced vibration response for a spring-mass system subjected to a harmonic response of the form $F(t) = F_0 \sin \omega t$.

$$x(t) = x_h(t) + x_p(t)$$

We know $x_h(t) = A_1 \cos(\omega_n t) + A_2 \sin(\omega_n t)$

Now we must find the undetermined coefficients for the forcing function

EOM $\ddot{x} + \omega_n^2 x = F_o \sin(\omega t)$

Trial solution

$$x_p(t) = Xsin(\omega t)$$

$$\ddot{x}_p(t) = -X\omega^2 \sin(\omega t)$$

Plug into the EOM

$$Xsin(\omega t)[-\omega^2 + \omega_n^2] = F_o \sin(\omega t)$$

$$X = \frac{F_0}{\omega_n^2 - \omega^2}$$

Plug into trial solution

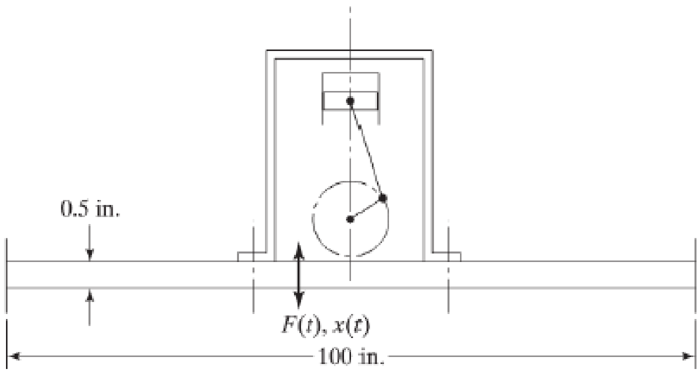
$$x_p(t) = \frac{F_0}{\omega_n^2 - \omega^2} \sin(\omega t)$$

General solution is therefore

$$x(t) = A_1 \cos(\omega_n t) + A_2 \sin(\omega_n t) + \frac{F_0}{\omega_n^2 - \omega^2} \sin(\omega t)$$

Problem 5

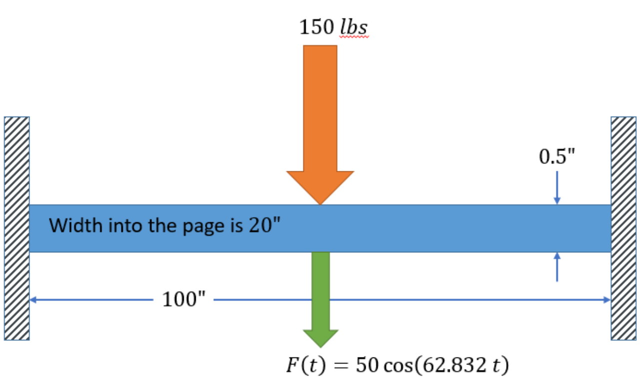
A reciprocating pump weighing 150 pounds is mounted at the middle of a steel plate of thickness 0.5 inches, width 20 inches, and length 100 inches as shown below, and is fixed at both ends. During operation of the pump, the plate is subjected to a harmonic force, $F(t) = 50\cos 62.832t$.



a. Determine the amplitude of vibration of the plate at steady-state.
b. Given the above information determine if the structure will fail (in this case, take failure as exceeding the yield strength of the material). Show clearly all work and calculations.
c. Based on your answer to part (b) determine the factor of safety of the structure.

a)

Draw FBD



$E_{steel} = 29\ _{E6}\ psi$

$I = \frac{1}{12} bh^3 = \frac{1}{12} 20(0.5)^3 = 0.208\ in^4$

$L^3 = 100^3 = 1{,}000{,}000\ in^3$

From table of stiffness constants for beams

$K = \frac{129EI}{L^3} = 1158 \frac{lb_f}{in}$

Find lb_m

The constant force of the motor on the steel plate is 150 lb_f we need to find the equivalent lb_m so we can use it in to find the ω_n

$$150\ lbf \left| \frac{1}{32.2} \frac{lb_m}{lbf} \right| \frac{1ft}{12\ in} = 0.388\ lb_m$$

Find ω_n

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1158}{0.388}} = 54.6$$

Use the EOM of a simple harmonic. We can do this because the system is said to be in steady state, therefore the damping must be canceled out or be negligable

$$x(t) = A_1 \sin(\omega_n t) + A_2 \cos(\omega_n t) + \frac{F_0}{\omega_n^2 - \omega^2} \cos(\omega t)$$

From the givin conditions, we can assume that $F_0 = 50$, $\omega = 62.832$, $x(0) = 0$, $v_0 = 0$

This means that

$$A_1 = 0$$

$$A_2 = -\frac{F_0}{\omega_n^2 - \omega}$$

The general solution is

$$x(t) = -\frac{F_0}{\omega_n^2 - \omega} \cos(\omega_n t) + \frac{F_0}{\omega_n^2 - \omega^2} \cos(\omega t)$$

$$x(t) = \frac{F_0}{\omega_n^2 - \omega} (\cos(\omega t) - \cos(\omega_n t))$$

Assuming that at some point $(\cos(\omega t) - \cos(\omega_n t)) = 1$

The max amplitude is therefore $\frac{F_0}{\omega_n^2 - \omega}$

Plug the knowns in to find the max displacement due to vibration

$$\delta_{vib} = \frac{F_0}{\omega_n^2 - \omega} = \frac{50}{54.6^2 - 62.832^2} = 0.133\ in$$

b) To find the max dispacement overall we must find the constant displacement due to the load of the motor

From a table of beam displacements, the beam with two ends fixed and a load in the midspan is

$$\delta_{load} = \frac{Pl^3}{192EI} = \frac{150 \cdot 100^3}{192 \cdot 29_{\ E6} \cdot 0.208} = 0.129\ in$$

The max displacement that the beam sees is the addition of both the constant displacement and the vibrational amplitude

$$\delta_{max} = \delta_{vib} + \delta_{load} = 0.133 + 0.129\ in$$

The max stress the beam sees is proportional to the max displacement by the equation

$\sigma = \delta \frac{E}{L} = 0.2625 \frac{29\,E6}{100} = 76{,}130\,psi$

Steel has a large range of yeild characteristics depending on the carbon content and post processing the range of yeild stress is

$$\sigma_y \in \{36{,}000\,,100{,}000\}\,psi$$

Therefore this beam can be designed not to fail.

c) The factor of safety is defined as $FOS = \frac{\sigma_y}{\sigma}$

For the low end steels

$$FOS = \frac{36}{76.130} = 0.4728$$

For high strngth steels

$$FOS = \frac{10}{76.130} = 1.313$$