Problem 1

A 100-kg machine is supported on an isolator of stiffness 700 x 10^3 N/m. The machine causes a vertical disturbance force of 350 N at a revolution of 3000 rpm. The damping ratio of the isolator is 0.2. Calculate (a) the amplitude of motion caused by the unbalanced force, (b) the force transmissibility ratio, and (c) the magnitude of the force transmitted to ground through the isolator.

$$m = 100\ [kg]$$
$$K = 700{,}000\ \left[\frac{N}{m}\right]$$
$$F_0 = 350\ [N]$$
$$\omega = 100\pi \left[\frac{rad}{s}\right] \leftarrow 3000 \left[\frac{rotation}{min}\right] \cdot \frac{1}{60} \left[\frac{min}{s}\right] \cdot 2\pi \left[\frac{rad}{rotation}\right]$$
$$\zeta = 0.2$$

a) Find the amplitude of motion caused by the unbalanced force
Need to find the natural frequency to calculate the frequency ratio

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{7000} = 26.6\pi$$

$$r = \frac{\omega}{\omega_n} = \frac{100\pi}{26.6\pi} = 3.75$$

The amplitude of the mass displacement caused by the disturbance after a transient period is

$$x_p(t) = \frac{F_0}{m((\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2)^{1/2}} = 3.79_{E-5}$$

b) The equation for transmissibility is

$$\frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right]^{\frac{1}{2}} = 0.1371$$

The force transmissibility is

$$\frac{F_T}{kY} = r^2 \frac{X}{Y} = 1.928$$

c) To find the force transmitted use the result from part b) and use the force transmissibility equation for a fixed base.

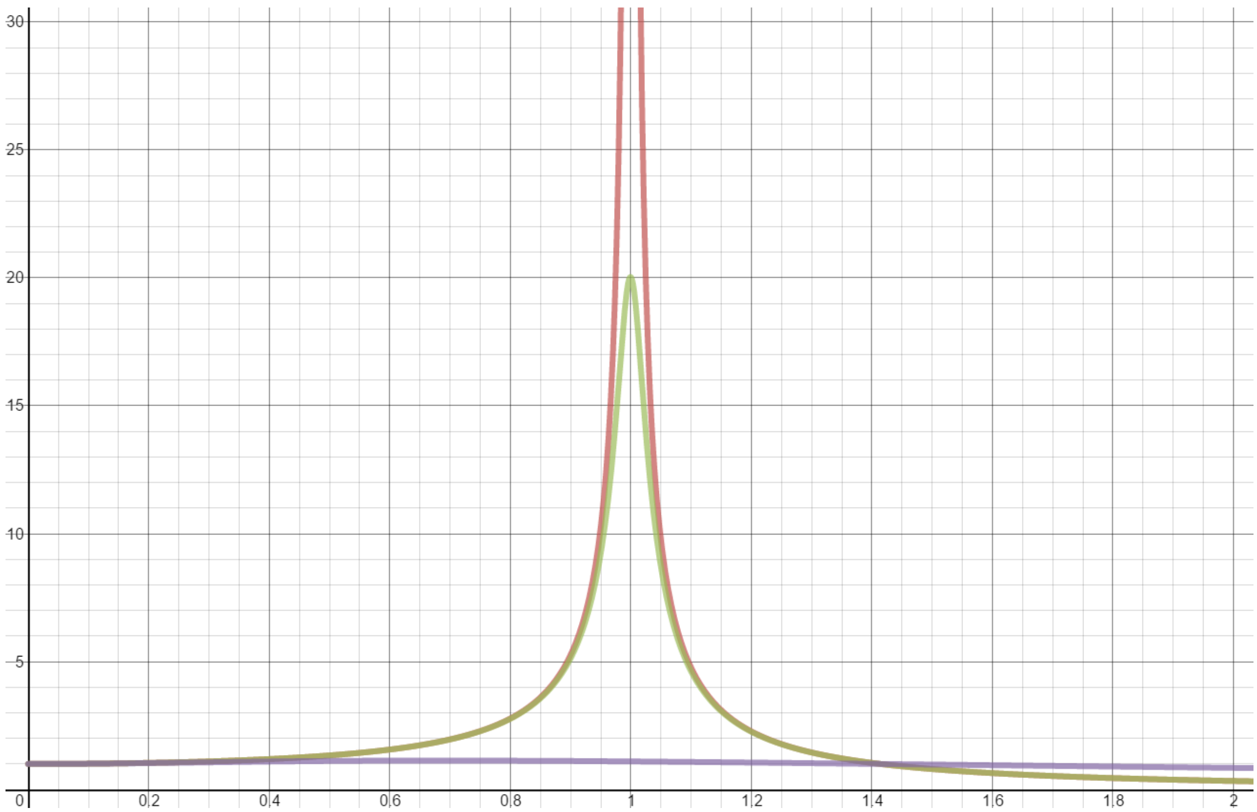
$$F_T = F_0 \frac{X}{Y} = 350 \cdot 0.1371 = 47.98\ [N]$$

Problem 2

Using MatLab, plot the transmissibility ratio of problem 1 for the cases of damping ratio equal to 0.001, 0.025, and 1.1.

All graphs are a function of ω

$$\zeta = 0.001$$



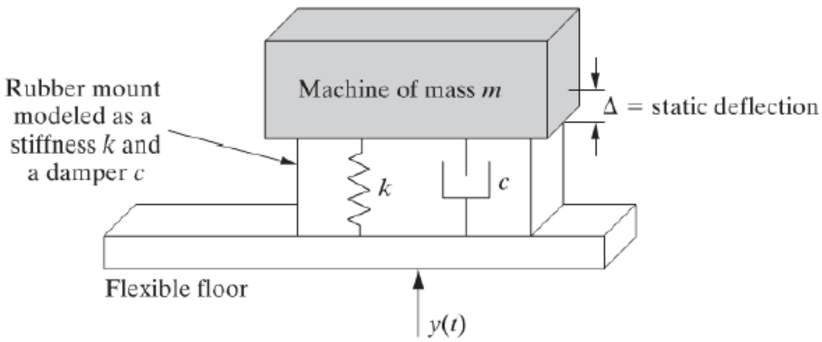
Red $\zeta = 0.001$ max @ $r = 1, \frac{X}{Y} = 500$

Green $\zeta = 0.025$ max @ $r = 1, \frac{X}{Y} = 20.03$

Purple $\zeta = 1.1$ max @ $r = 0.685, \frac{X}{Y} = 1.13$

Problem 3

A machine weighing 2000 N rests on a support, as illustrated below. The support deflects about 5 cm as a result of the weight of the machine. The floor under the support is somewhat flexible and moves, because of the motion of a nearby machine, harmonically near resonance (r= 1) with an amplitude of 0.2 cm. Model the floor as base motion, assume a damping ratio of ς = 0.01, and calculate the transmitted force and the amplitude of the transmitted displacement.



$$m = 2000\ [N] \cdot \frac{1}{9.81}\left[\frac{s^2}{m}\right] = 203.8\ [kg]$$
$$\delta = 5_{E-2}\ [m]$$
$$r = 1$$
$$Y = 0.2_{E-2}\ [m]$$
$$\zeta = 0.01$$

K can be found from the relationship between the initial force and static displacement

$$k = \frac{F_0}{\delta} = \frac{2000}{0.05} = 40{,}000\frac{kg}{s^2}$$

$$F_T = kYr^2\left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right]^{\frac{1}{2}} = 40{,}000 \cdot 0.2_{E-2}\left[\frac{1 + (2 \cdot 0.01)^2}{(2 \cdot 0.01)^2}\right]^{\frac{1}{2}} = 4{,}008\ [N]$$

$$\frac{X}{Y} = 50.01$$

The amplitude of the transmitted displacement is

$$X = 50.01 \cdot 0.2_{E-2} = 0.1\ [m]$$

Problem 4

A very common example of base motion is the single-degree-of-freedom model of an automobile driving over a rough road. The road is modeled as providing a base motion displacement of $y(t)$= (0.01) sin(5.818t) m. The suspension provides an equivalent stiffness of k=4x10^5 N/m, a damping coefficient of c=40x10^3 kg/s, and a mass of 1007 kg. Determine the amplitude of the absolute displacement of the automobile mass.

$$x = Y\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{\frac{1}{2}}$$

$$\omega = 5.818$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4_{E5}}{1007}} = 19.93$$

$$r = 0.2919$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{40_{E3}}{2\cdot 1007\cdot 19.93} = 0.9965$$

$$x = 0.01\left[\frac{1+(0.5818)^2}{(0.9148)^2+(0.5818)^2}\right]^{1/2} = 1.0671\ [cm]$$

Problem 5

A system modeled by a spring-mass-damper with moving base has a mass of 225 kg with a spring stiffness of 3.5x10^4 N/m. Calculate the damping coefficient given that the system has a deflection (X) of 0.7 cm when driven at its natural frequency while the base amplitude (Y) is measured to be 0.3 cm.

$$m = 225\ [kg] \qquad Y = 0.3_{E-2}\ [m]$$

$$k = 3.5_{E4}\left[\frac{N}{m}\right] \qquad r = 1$$

$$X = 0.7_{E-2}\ [m] \qquad \frac{X}{Y} = \frac{0.7}{0.3} = 2.333$$

$$\frac{X}{Y} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{\frac{1}{2}} = 2.333$$

Solve for ζ

$$\frac{1+4\zeta^2}{4\zeta^2} = 2.333^2 \quad \rightarrow \quad \frac{1}{2.333^2-1} = 4\zeta^2$$

$$\zeta = \sqrt{\frac{1}{4(2.333^2-1)}} = 0.2372$$

$$c = 2\zeta\sqrt{k\cdot m} = 1331.28$$